

## **ABSTRACT**

**of the dissertation on the topic: "Mathematical modeling and research of boundary value problems for vibrations of the flat element" for the degree of Doctor of Philosophy (PhD) in the specialty "6D060100-Mathematics"**  
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### **Purpose and common characteristics of the dissertation work**

Plates as flat structural elements are widely used in the construction and engineering industries. This can be explained by the ease of thin-layer structures, the efficiency and cheapness of forms.

The problem of creating structures made of waterproof materials and, in addition, production problems arising in the field of geophysics, seismology, require the creation of a new refined theory of calculations.

These requirements are necessary in order to take into account their mathematical model, mechanical properties, and the influence of forces during deformation as realistically as possible during the operation of structures. This problem is associated with the design of many engineering structures and structures that have been designed recently, with the laying of the foundations of various buildings, airfield flight pads and highways.

The main theoretical problem in this area is the study of boundary value problems of vibrations of plane elements and the dynamics of materials, as well as the improvement of strict mathematical methods for solving two-dimensional and three-dimensional problems, but this problem has not yet been completely solved, since the computational methods currently used do not provide exhaustive answers to questions arising in boundary value problems of vibrations. At the same time, in this study there is no unambiguous approach to which mathematical model should be taken as a basis.

In modern science, materials with viscoelastic properties, in particular polymer materials, are becoming increasingly widespread, fundamental research in the field of non-stationary deformation processes of viscoelastic bodies and specific calculations of elements of engineering structures made of viscoelastic materials are widely used in various fields of engineering practice.

This includes the tasks of determining strength, evaluating reliability and durability, determining frequency characteristics, choosing optimal parameters that provide effective operating conditions, stability, and some other issues related to the behavior of structural elements under the action of dynamic influences on them. These studies have a wide range of relevant applications in such fields of science and technology as seismology, geophysics, acoustic flaw detection, mechanical engineering, space technology.

An actual problem of the modern stage of theoretical research in the field of non-stationary vibrations of viscoelastic bodies, along with the development of new models of dynamic deformation of viscoelastic materials close to experimental ones, is the development of effective mathematical methods for the study of many classes of plane and spatial problems within the framework of

known models, theoretical analysis of the main mechanical factors due to the influence of viscoelastic parameters.

Despite a large number of theoretical and applied research in this field, a general overview of which is presented in the first section, and on specific issues in the relevant chapters and sections of the dissertation work, the problems of solving many important classes of boundary value problems and their analysis remain mostly open or require further refined development.

Among them are the problems of non-stationary vibrations of rods, plates and shells, taking into account rheology. When solving problems of this class, approximate vibration equations obtained from the three-dimensional equations of motion of the theory of elasticity with the help of various hypotheses and prerequisites of a mechanical or geometric nature are used as the main resolving equations, which significantly simplify the solutions of the problem.

In addition, the initial three-dimensional problem of the theory of elasticity is reduced to a two-dimensional or one-dimensional one using various mathematical methods, including such methods as variational, asymptotic, power series method, etc.

To date, a huge number of studies have been carried out on bringing a three-dimensional problem to a two-dimensional one by engineering and mathematical methods. But these studies do not completely exhaust the problem. The solution of this problem for bodies with different geometries continues to this day, as evidenced by the publications of domestic, Russian and foreign scientists.

Adjacent to them is the problem of studying the dynamic behavior of a material interacting with a deformable medium on the basis of vibration equations derived using a strict mathematical apparatus. The area of application of such tasks is quite wide, because the rods are elements of many engineering structures, ranging from the simplest machines, devices and structures, to the most complex space technology, nuclear and hydroelectric power plants, shipbuilding, etc. Taking into account the rheological properties and anisotropy of the shell material of the interacting medium, temperature changes, thickness variability and other factors leads to a significant complication in the study of these problems. On the other hand, the correct consideration of these factors is of great importance for ensuring the strength, reliability and durability of structures, allow you to significantly save material resources.

### **Relevance of the dissertation work**

The use of qualitatively new materials and technologies that meet the modern level of scientific and technological progress puts forward increased requirements for the study of the non-stationary behavior of elements of various structures, taking into account rheology. In particular, the elements of modern buildings and structures include ground and underground elements such as foundations, which have a wide range of mechanical characteristics and geometric parameters.

The large scale of housing construction in Kazakhstan leads to the need for further development and improvement of calculation methods in construction mechanics, hydrodynamics and in the field of geophysics.

When conducting research in specific engineering problems and in modern problems of vibrations, it becomes necessary to take into account the physical and mechanical properties of plane elements in more detail.

One of the main issues is mathematical modeling and the study of boundary value problems of vibrations of plane elements. The relevance of this topic has been noted at numerous congresses and symposiums. The large scale of housing construction in Kazakhstan leads to the need for further development and improvement of calculation methods in construction mechanics, hydrodynamics and in the field of geophysics.

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### **Reliability and validity of the research results**

The validity of the provisions and conclusions of the dissertation work are based on the consideration of a flat element in a three-dimensional formulation, using well-tested analytical and numerical methods of mathematics.

In a brief introduction and a review of the literature, it is not possible to answer all the most important and valuable things that have been achieved by research in solving the problem under consideration. This is primarily due to the fact that the content of works in the field of mathematics and mechanics is determined not only by the accepted model of the foundation, but also by the type of structures, and what problem is being considered: statistical or dynamic, linear or nonlinear, etc.

Operational or symbolic calculus or integral transformation, the first versions of which were constructed by Oliver Heaviside, an English scientist, engineer, mathematician and physicist, then by Laplace-Carson, have a further continuation in science, such as the two-fold integral Laplace-Carson transformations, which are best described in the form of tabular integrals and are given in a voluminous monograph by Vitaly Arsenyevich Ditkin and Anatoly Platonovich Prudnikov.

Later, a variant was constructed for differential equations with periodic coefficients by Kim Galyamovich Valeev, a student of Academician Chitaev (Chebyshev).

The subsequent construction was carried out by the English specialist D. R. Bland, the author of the monographs "The theory of linear viscoelasticity". The monograph of the English scientist describes the theory of linear viscoelastic media with an infinite relaxation spectrum. Wave propagation is analyzed and the solution of a number of quasi-static problems is given. The possibility of a proper selection of a viscoelastic model based on experimental data and "Nonlinear dynamic theory of elasticity" is considered. The paper provides a brief summary of the analysis of large deformations and stresses, the defining equations and the propagation of shock waves. Adiabatic and isentropic approximations of the general problem and types of possible discontinuities in isotropic compressible and incompressible media are considered. The book is distinguished by the

accessibility of the presentation, the predominance of the physical approach over the formally mathematical one.

The main advantage of this integral method (transformation) is that its application leads to fairly complex equations written in functionals to fairly simple equations.

The fundamental results of domestic and foreign scientists were obtained in the studied area. Therefore, here we will only mention some of the main works, which are based on the most common models of the mechanics of deformable solids.

For this reason, the proposed brief review does not pretend to cover all the available results that are directly relevant to this work. We will refer to works that are only close to the issues and problems addressed in this dissertation work, and are of fundamental importance in mathematics and mechanics.

Fundamental ideas and approaches in the development of mathematical models, theoretical and experimental research in the field of dynamic interaction of the plate are associated with the names of such scientists as Zh.D.Achenbach, E.I.Grigolyuk, G.I.Petrashen, H.A.Rakhmatullin, S.P. Timoshenko, I.G. Filippov, A.N.Tyurekhodzhaev, M.I. Ramazanov, B.D. Dzhanmuldaev, A.Zh.Seitmuratov and many others.

The problems of wave propagation in elastic and viscoelastic media were studied in the works of scientists G.Kolsky, E.I. Grigolyuk, Yu.N. Rabotnov, H.A.Rakhmatullin, S.P.Timoshenko, I.G.Filippov, A.Zh.Seitmuratov and many others.

Many topical scientific and technical problems are associated with the study of vibrational processes and wave propagation in continuous media. The use of the results of these studies is of great benefit when considering non-stationary vibrational and wave processes. However, a number of questions arise related to the reaction of the medium to external influences, the methods of excitation of movements, the kinematic characteristics of waves, the geometry of bodies, the solution of which is of applied importance and is achieved using its own methods typical for this field.

Summarizing the given brief review of the works, which is certainly not complete, it can be noted that the solution of dynamic problems of flat elements and the study of boundary value problems of vibrations of a flat element in the form of plates is far from complete. When studying most of them, simplifying assumptions were made about the viscous properties of plate materials, and only a limited range of oscillation frequencies was considered.

### **The purpose of the dissertation work**

Construction of methods for calculating non-stationary vibrations taking into account the boundary value problems of vibrations of flat elements located on a deformable base and mathematical modeling and research.

The scientific significance of this study is to consider the bends of flat elements in general and to consider in more detail the influence of temperature anisotropy. The results obtained in the course of this research work can be used to calculate the vibrations of foundations and calculate the movements of underground structures.

**The object and subject of the study** are the theory of vibrations, a mathematical model of vibrations of plane elements.

### **Research methods**

When studying wave fields and linear deformable media or when solving problems of vibration of plates and rods, depending on the formulation of the problem, certain mathematical methods are used such as

- Integral transforms Fourier and Laplace
- Formulas for approximate inversion of Laplace transformations
- The method of separation of variables
- The method of plane waves
- Series method

### **Scientific significance**

The scientific significance of the dissertation work is as follows:

1. When setting the main boundary value problems of the dynamics of elastic viscoelastic media with small deformations, the stress-strain state of the entire body at any point was determined.

2. The nonlinear dependences between the average stress and the average strain of small deformations for a viscoelastic body are determined.

3. Simple models of real viscous bodies with a linear integral are obtained, initially showing the properties of instantaneous elasticity for the motion of the medium depending on the structure and geometry.

4. The problem of modeling small deviations in an elastically deformable medium using a perforated porous system filled with a liquid or gas.

5. Based on the dynamics of the deformable medium, taking into account geometric and mechanical factors, various boundary conditions for vibrations of a flat rectangular element in a three-dimensional linear representation are obtained.

6. Solving the integro-differential equations for the problem of boundary vibrations of the plate, the values characterizing the displacement of the tangent points for power series that converge on the arguments of hyperbolic functions are determined.

7. The rationality of the approximate decomposition method for solving the equations of motion of a deformable medium is shown.

8. The equation of the frequency of free vibrations of a flat structural element fixed by a hinge interacting with a deformable medium is solved.

9. Transcendental frequency equations are reduced to algebraic equations and the influence of boundary conditions on the edges of a rectangular plate or a rectangular flat element, as well as geometric and mechanical parameters on the natural oscillation frequencies of a rectangular plane. From previous results, it was shown that Phillipov and recent results give a positive solution.

### **Practical significance of the dissertation work**

New methods of calculation on a deformable basis and deformable flat elements are shown, taking into account the influence of the above factors. The method of calculating the frequency of free vibrations of flat elements at various fixing of their boundaries is also shown.

The validity of the conclusions and results presented in the dissertation work is based on the use of certain analytical methods that consider a flat element in a three-dimensional representation.

### **Approbation of the work**

The main provisions of the dissertation and the scientific results presented in it were reported and discussed at international, scientific, scientific-practical conferences and scientific seminars of the department. According to the results of the dissertation, reports were made at international conferences and at conferences of foreign countries: "Modern Mathematics: problems and applications" (Kyzylorda, 2017); "Differential Equations and related problems" (2018, Sterlitamak, Russia); "Theoretical and applied issues of Mathematics, Mechanics and Computer Science", (June 12-13. - Karaganda, 2019); "Actual problems of analysis, differential equations and algebra (EMJ-2019) (Nur-Sultan, 2019); at a seminar led by Professor M.I. Ramazanov (E.A. Buketov KarSU); at a seminar of the Department "Mathematical Analysis and Differential Equations" of the KarSU named after Academician Buketov.

### **Publications**

According to the results of the dissertation, 13 works were published. Including: 3 articles in publications recommended by the Committee for Control in the Field of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan, in a scientific journal with a non-zero impact factor included in the Scopus database - 1, 1 work in foreign publications, 4 works in the proceedings of international scientific conferences and collections of abstracts, publications in other publications - 4.

### **The structure of the dissertation**

The dissertation work consists of an introduction, two sections, a conclusion, and a list of references. The numbering of formulas in sections is three - digit, the first number means the number of the section, the second - the number of the subsection, the third - the own number of the formula within the subsection.